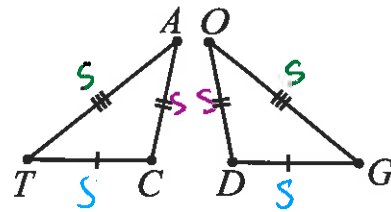


## Proving Triangles Congruent

There are only 5 ways to prove two triangles are congruent!

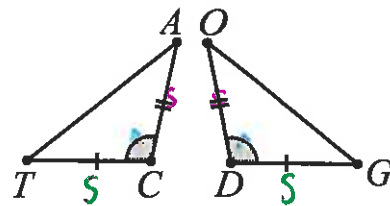
1. SSS: Side-Side-Side

$$\left. \begin{array}{l} \text{Side } \overline{CT} \cong \overline{GD} \\ \text{Side } \overline{AC} \cong \overline{OD} \\ \text{Side } \overline{AT} \cong \overline{OG} \end{array} \right\} \Delta CAT \cong \Delta DOG \text{ By SSS}$$



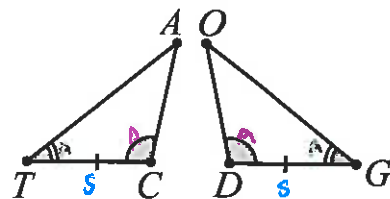
2. SAS: Side-Angle-Side

$$\left. \begin{array}{l} \text{Side } \overline{TC} \cong \overline{GD} \\ \text{Angle } \angle C \cong \angle D \\ \text{Side } \overline{AC} \cong \overline{OD} \end{array} \right\} \Delta CAT \cong \Delta DOG \text{ By SAS}$$



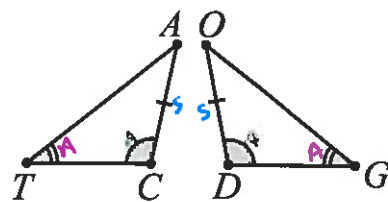
3. ASA: Angle-Side-Angle

$$\left. \begin{array}{l} \text{Angle } \angle T \cong \angle G \\ \text{Side } \overline{TC} \cong \overline{GD} \\ \text{Angle } \angle C \cong \angle D \end{array} \right\} \Delta CAT \cong \Delta DOG \text{ By ASA}$$



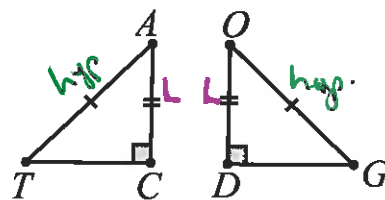
4. AAS: Angle-Angle-Side

$$\left. \begin{array}{l} \text{Angle } \angle T \cong \angle G \\ \text{Angle } \angle C \cong \angle D \\ \text{Side } \overline{AC} \cong \overline{OD} \end{array} \right\} \Delta CAT \cong \Delta DOG \text{ By AAS}$$



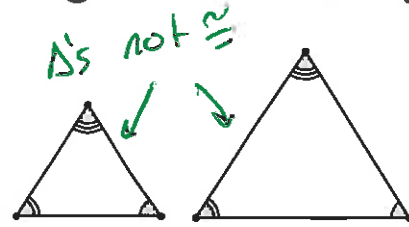
5. HL: Hypotenuse-Leg (rt Δ's)

$$\left. \begin{array}{l} \text{Hyp. } \overline{AT} \cong \overline{OG} \\ \text{Leg. } \overline{AC} \cong \overline{OD} \end{array} \right\} \Delta CAT \cong \Delta DOG \text{ By HL}$$

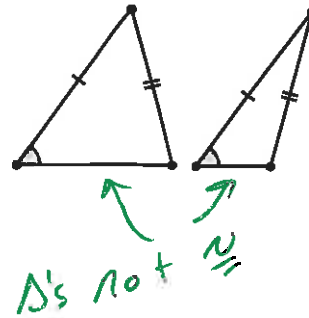


# Methods that **DO NOT** prove Triangles to be Congruent

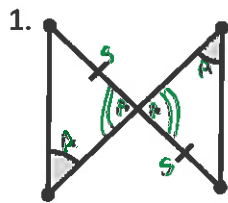
1. AAA: Angle-Angle-Angle



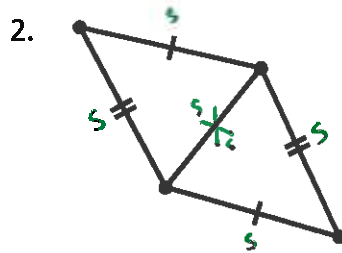
2. ASS or SSA: Angle-Side-Side



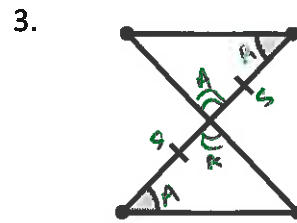
Examples: Which triangle postulate shows that the triangles are congruent?



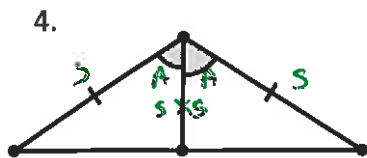
AAS



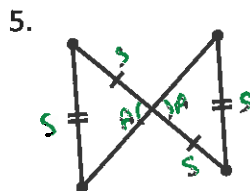
SSS



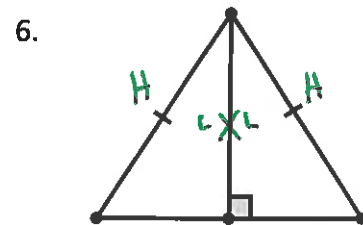
ASA



SAS



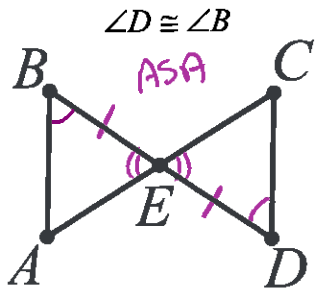
Δ's not  $\cong$   
(ASS)



H-L

Examples:

1. Given: E midpoint of  $\overline{BD}$



**Congruent Parts**  
 A  $\angle B \cong \angle D$   
 S  $\overline{BE} \cong \overline{DE}$   
 A  $\angle BEA \cong \angle DEC$

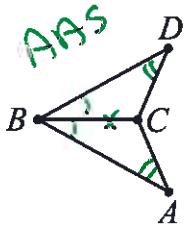
**Reason**  
 Given  
 midpt makes 2  $\cong$  segs.  
 vert.  $\angle$ 's are  $\cong$   
 ASA

Name the Congruent Triangles:  $\triangle BEA \cong \triangle DEC$

Rigid motion: Rotate  $\triangle ABE$   $180^\circ$  counter clock wise so  $\triangle ABE$  maps to  $\triangle CDE$ .

2. Given:  $\overline{BC}$  bisects  $\angle ABD$

$\angle A \cong \angle D$



**Congruent Parts**  
 A  $\angle A \cong \angle D$   
 A  $\angle CBD \cong \angle CBA$   
 S  $\overline{BC} \cong \overline{BC}$

**Reason**  
 Given  
 $\angle$  bisector  $\div$   $\angle$  into 2  $\cong$   $\angle$ 's.  
 Reflexive.  
 AAS

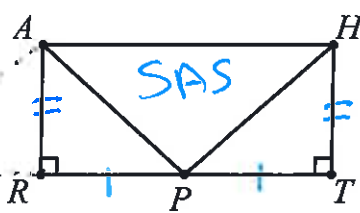
Name the Congruent Triangles:  $\triangle CBD \cong \triangle CBA$

Rigid motion: Reflect  $\triangle ABC$  over line  $\overline{CB}$  so  $\triangle ABC$  maps to  $\triangle DBC$ .

3. Given: P midpoint of  $\overline{RT}$

$\angle R$  &  $\angle T$  right angles

$\overline{AR} \cong \overline{HT}$



**Congruent Parts**  
 S  $\overline{AR} \cong \overline{HT}$   
 A  $\angle R \cong \angle T$   
 S  $\overline{RP} \cong \overline{TP}$

**Reason**  
 Given  
 Rt  $\angle$ 's are  $\cong$   
 midpt make 2  $\cong$  segs.  
 SAS.

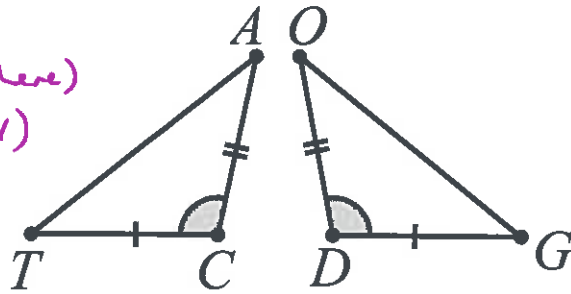
Name the Congruent Triangles:  $\triangle ARP \cong \triangle HTP$

Rigid motion: Reflect  $\triangle PTH$  over the  $\perp$  bisector of  $\overline{RT}$  so  $\triangle PTH$  maps to  $\triangle PRA$ .

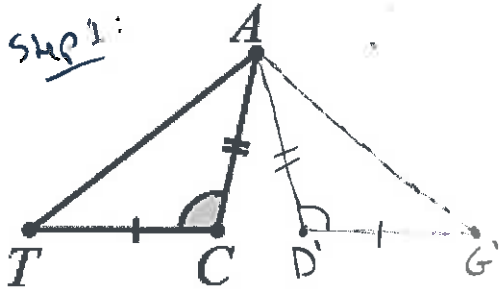
OR Translate  $\triangle PTH$  along vector  $\overrightarrow{TR}$  so  $\overline{TH}$  maps to  $\overline{RA}$ , forming  $\triangle P'RA$ .  
 reflect  $\triangle P'RA$  over  $\overline{RA}$  so that  $\triangle P'RA$  maps to  $\triangle PRA$ .

Justify by rigid motions that SAS is true:

What? (is being moved)  
 Where? (is it going)  
 How? (is it getting there)  
 Why? (is  $\cong$  preserved)

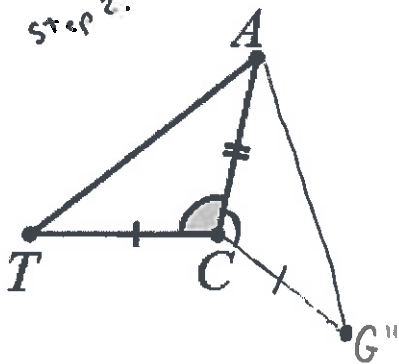


Step 1:



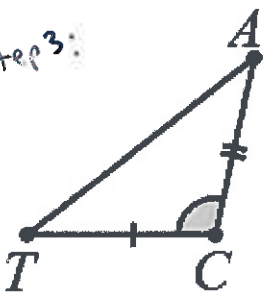
Translate  $\triangle DOG$  along vector  $\vec{OA}$   
 So O maps to A forming  $\triangle DAG$ .  
 where

Step 2:



Rotate  $\triangle D'AG'$  clockwise by  $m\angle D'AC$   
 So  $D'$  maps to  $C$ , forming  $\triangle CAG''$  ← where  
 $D'$  maps to  $C$  because  $\overline{AD'} \cong \overline{AC}$  and  
 Rotation preserves seg. length. ← why.

Step 3:



Reflect  $\triangle CAG''$  over  $\overline{AC}$   
 So  $G''$  maps to  $T$ , forming  $\triangle CAT$ . } where  
 $G''$  maps to  $T$  because  $\angle ACG'' \cong \angle ACT$   
 and  $\overline{CG''} \cong \overline{CT}$ . Reflection preserves  
 distance and angle measure. } why.